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EXACT AND APPROXIMATESOLUTIONS OF TWO PHASE INVERSE STEFAN PROBLEM

Abstract. The main idea is finding the exact solution of two phase inverse Stefan problem for degenerate domain with moving boundary.

Tracking answers of these questions will be organized as following. For finding analytical solution we mainly follow the method proposed by S.N. Kharin applying Faa Di Bruno's formula for Integral Error Functions. In the continuation of this section Integral Error Functions and its properties necessary for elaboration of new methods are presented.

The Stefan problem is very complicated for exact solution. Therefore we try to find an approximate solution by profile assignment of the temperature with change of differential equation by heat balance equations, which are obtained by integration with respect to x and t. For finding approximate solution we mainly follow the method proposed by S.N. Kharin applying Integral Power Balance Method.

Keywords: Integral Error Functions, Integral Power Balance Method, Two Phase Inverse Stefan Problem.

Problem formulation

In the following two sections we will deal to find theexact and approximate solutions of heat equations:

$$\frac{\partial u_1}{\partial t} = a_1^2 \frac{\partial^2 u_1}{\partial x^2}, \quad 0 < x < \alpha(t), \quad t > 0 \tag{1}$$

$$\frac{\partial u_2}{\partial t} = a_2^2 \frac{\partial^2 u_2}{\partial x^2}, \quad \alpha(t) < x < \infty, \quad t > 0 \tag{2}$$

Subjected to the following conditions:

$$u_1(0,0) = 0 \tag{3}$$

$$u_2(x,0) = f(x) \quad \alpha(t) < x < \infty \tag{4}$$

$$-\lambda_1 \frac{\partial u_1}{\partial x}\bigg|_{x=0} = P(t), \quad t > 0 \tag{5}$$

$$u_1(\alpha(t),t) = u_2(\alpha(t),t) = u_m \tag{6}$$

The Stefan's condition:

$$-\lambda_1 \frac{\partial u_1}{\partial x}\bigg|_{x=\alpha(t)} = -\lambda_2 \frac{\partial u_2}{\partial x}\bigg|_{x=\alpha(t)} + L\gamma \frac{d(\alpha(t))}{dt} \tag{7}$$

$$u_2\,|_{x=\infty} = 0 \tag{8}$$

It is necessary to find temperature distribution $u_1(x,t)$ and $u_2(x,t)$ also it is required to reconstruct the boundary function $P(t)$ if the free boundary $\alpha(t)$ is given. Such problem is called two phase inverse Stefan Problem. The heat spread in the solid is negligible because of the physical properties of contact material. This condition is valid for refractory metals like wolfram. After getting the solutions we try to discuss given both solutions.

I. EXACT SOLUTION OF TWO PHASE INVERSE STEFAN PROBLEM

1.1. Introduction

The first analytical solution of two phase inverse Stefan problem, which describes the dynamics of soil freezing has been published by Lame and Clayperon.

Despite the quite extensive list of problems in literature which lead to the necessity to solve Stefan type problems see: e.g., [1–9] and a long bibliography [9] on methods for solving these problems lead to additional difficulties which occur due to the degeneracy of domains. In some specific cases particularly for free moving boundaries it is possible to construct Heat potentials and a problem can be reduced to the system of integral equations [2], [3], however in the case of degeneracy, singularity in integral equations occur, and method of successive approximations is inapplicable in general. Moreover, the use of numerical methods is problematic when the number of parameters is great. Therefore, development of new analytical methods is very important especially for various applications because it enables one to analyze an interrelationship of different input parameters and their influence on the dynamics of investigating phenomena.

As for applications: a wide range of electric contact phenomena, in particular, the phenomena occurring at the interaction of electrical arc with electrode can be described in dynamic use of the presented method see e.g., [9] for very short arc duration (nanosecond diapason), when experimental investigation is very difficult. In this study we will try to find solution of two Phase Inverse Stefan problem for degenerate domain with moving boundary.

1.2. Problem Solution

Let us consider this problem at the suggestion that $\alpha(t)=\sum_{n=1}^{\infty}\alpha_n t^{n/2}$ is given.

The unknown functions are $u_1(x,t), u_2(x,t)$, $P(t)$ we represent in the form of Heat polynomials and Integral Error Functions:

$$u_1(x,t)=\sum_{n=0}^{\infty}\left[\left(2a_1\sqrt{t}\right)^{2n} A_{2n} i^{2n}erfc\left(\frac{x}{2a_1\sqrt{t}}\right)+\left(2a_1\sqrt{t}\right)^{2n+1} A_{2n+1} i^{2n+1}erfc\left(-\frac{x}{2a_1\sqrt{t}}\right)\right] \quad (9)$$

$$u_2(x,t)=\sum_{n=0}^{\infty}\left[\left(2a_2\sqrt{t}\right)^{2n} B_{2n} i^{2n}erfc\left(\frac{x}{2a_2\sqrt{t}}\right)+\left(2a_2\sqrt{t}\right)^{2n+1} B_{2n+1} i^{2n+1}erfc\left(-\frac{x}{2a_2\sqrt{t}}\right)\right] \quad (10)$$

$$P(t)=\sum_{n=0}^{\infty} P_n t^{\frac{n}{2}} \quad (11)$$

Where the coefficients $A_{2k}, A_{2k+1}, B_{2k}, B_{2k+1}, P(t)$ should be found satisfying the boundary conditions.

Let us satisfy first the initial condition (4). It is clear that:

$$u_2(x,0)=\lim_{t\to 0}u_2(x,t)=\sum_{n=0}^{\infty}\left[\lim_{t\to 0}\left(2a_2\sqrt{t}\right)^{2n}B_{2n}i^{2n}erfc\left(\frac{x}{2a_2\sqrt{t}}\right)+\right.$$
$$\left.+\lim_{t\to 0}\left(2a_2\sqrt{t}\right)^{2n+1}B_{2n+1}i^{2n+1}erfc\left(-\frac{x}{2a_2\sqrt{t}}\right)\right] \tag{12}$$

by properties of Integral Error Functions:

$$\lim_{x\to\infty}\frac{i^n erfc(x)}{x^n}=0$$

by using L'Hospital Rule to find limit:

$$\lim_{x\to\infty}\frac{i^n erfc(-x)}{x^n}=\lim_{x\to\infty}\frac{i^{n-1}erfc(-x)}{nx^{n-1}}=\lim_{x\to\infty}\frac{i^{n-2}erfc(-x)}{n(n-1)x^{n-2}}=\ldots=\lim_{x\to\infty}\frac{erfc(-x)}{n!}=\frac{2}{n!}$$

So therefore,

$$u_2(x,0)=\sum_{n=0}^{\infty}B_{2n+1}\frac{2}{(2n+1)!}x^{2n+1}=f(x) \tag{13}$$

If we expand $f(x)$ into Taylor series:

$$f(x)=\sum_{n=0}^{\infty}\frac{f^{(2n+1)}(0)}{(2n+1)!}x^{2n+1} \tag{14}$$

$$\sum_{n=0}^{\infty}B_{2n+1}\frac{2}{(2n+1)!}x^{2n+1}=\sum_{n=0}^{\infty}\frac{f^{2n+1}(0)}{(2n+1)!}x^{2n+1}\quad B_{2n+1}=\frac{1}{2}f^{(2n+1)}(0),\quad (n=0,1,2,\ldots) \tag{15}$$

From the condition (6) for $u_1(\alpha(t),t)=u_m$:

$$u_2(x,t)=\sum_{n=0}^{\infty}\left[\left(2a_2\sqrt{t}\right)^{2n}B_{2n}i^{2n}erfc\left(\frac{x}{2a_2\sqrt{t}}\right)+\left(2a_2\sqrt{t}\right)^{2n+1}B_{2n+1}i^{2n+1}erfc\left(-\frac{x}{2a_2\sqrt{t}}\right)\right]\Bigg|_{x=\alpha(t)}=u_m$$

The main problem is to find the remaining unknown coefficients we have, also if $\sqrt{t}=\tau$, then our expression will get:

$$\frac{\partial^{2k}}{\partial\tau^{2k}}\left(\sum_{n=0}^{\infty}\left[\left(2a_2\tau\right)^{2n}B_{2n}i^{2n}erfc\left(\frac{\alpha(\tau^2)}{2a_2\tau}\right)+\left(2a_2\tau\right)^{2n+1}B_{2n+1}i^{2n+1}erfc\left(-\frac{\alpha(\tau^2)}{2a_2\tau}\right)\right]\right)=\frac{\partial^{2k}}{\partial\tau^{2k}}(u_m)=0$$

$$\sum_{n=0}^{\infty}\sum_{l=0}^{2k}\left(2a_2\right)^{2n}B_{2n}\binom{2k}{l}\frac{(2n)!}{(2n-2k+l)!}\cdot\tau^{2n-2k+l}\cdot\frac{\partial^{2k}}{\partial\tau^{2k}}\left(i^{2n}erfc\left(\frac{\alpha(\tau^2)}{2a_2\tau}\right)\right)+$$
$$+\left(2a_2\right)^{2n+1}B_{2n+1}\binom{2k}{l}\frac{(2n)!}{(2n+1-2k+l)!}\cdot\tau^{2n+1-2k+l}\cdot\frac{\partial^{2k}}{\partial\tau^{2k}}\left(i^{2n+1}erfc\left(-\frac{\alpha(\tau^2)}{2a_2\tau}\right)\right)\Bigg]=0$$

After taking both parts 2k-derivatrives we can see following expression:

$$\sum_{n=0}^{\infty}\left(2a_2\right)^{2n}B_{2n}\frac{(2k)!}{(2k-2n)!}\frac{\partial^{2k-2n}}{\partial\tau^{2k-2n}}\left(i^{2n}erfc\left(\frac{\alpha(\tau^2)}{2a_2\tau}\right)\right)\Bigg|_{\tau=0}+$$

$$+\left(2a_2\right)^{2n+1} B_{2n+1} \frac{(2k)!}{(2k-2n-1)!} \frac{\partial^{2k-2n-1}}{\partial \tau^{2k-2n-1}} \left(i^{2n+1} erfc\left(-\frac{\alpha(\tau^2)}{2a_2\tau} \right) \right)\Bigg|_{\tau=0} = 0$$

For this purpose we use the Faa Di Bruno formula (Arbogast formula) for a derivative of a composite function:

$$\frac{\partial^{2k-2n}}{\partial \tau^{2k-2n}} \left(i^{2n} erfc\left(\frac{\alpha(\tau^2)}{2a_2\tau} \right) \right) = \sum_{j=0}^{2k-2n} i^{2n} erfc^{(j)}(\delta) \beta_{2k-2n,j}(\delta', \delta'', ..., \delta^{2k-2n-j})$$

$$i^{2n} erfc^{(j)}(\delta) = \frac{d^j}{d\tau^j} \left(i^{2n} erfc\left(\delta\right) \right)\Big|_{\tau=0} = (-1)^{j-2n-1} \frac{2}{\sqrt{\pi}} H_{j-2n-1}(\delta) \exp(-\delta^2)$$

$$\frac{\partial^{2k-2n-1}}{\partial \tau^{2k-2n-1}} \left(i^{2n+1} erfc\left(\frac{\alpha(\tau^2)}{2a_2\tau} \right) \right) = \sum_{j=0}^{2k-2n-1} i^{2n+1} erfc^{(j)}(\delta) \beta_{2k-2n-1,j}(\delta', \delta'', ..., \delta^{2k-2n-1-j})$$

$$i^{2n+1} erfc^{(j)}(\delta) = \frac{d^j}{d\tau^j} \left(i^{2n+1} erfc\left(\delta\right) \right)\Big|_{\tau=0} = (-1)^{j-2n-2} \frac{2}{\sqrt{\pi}} H_{j-2n-2}(\delta) \exp(-\delta^2)$$

Let us denote our boundaries in the following form to get rid of the singularity:

$$\gamma(t) = \alpha(t) - \alpha_0 = \gamma_1 t^{1/2} + \gamma_2 t + ... = \sum_{n=0}^{\infty} \gamma_{n+1} t^{(n+1)/2}$$

$$\alpha(\tau^2) = \gamma_1 \tau + \gamma_2 \tau^2 + ... + \gamma_{n+1} \tau^{n+1} = \sum_{n=0}^{\infty} \gamma_{n+1} \tau^{(n+1)}$$

Then we can easily find our boundaries:

$$\delta\big|_{\tau=0} = \frac{\sum_{n=0}^{\infty} \gamma_{n+1} \tau^{n+1}}{2a_2\tau}\Bigg|_{\tau=0} = \frac{\gamma_1}{2a_2}$$

$$\delta\big|_{\tau=0} = -\frac{\sum_{n=0}^{\infty} \gamma_{n+1} \tau^{n+1}}{2a_2\tau}\Bigg|_{\tau=0} = -\frac{\gamma_1}{2a_2}$$

Finally, we get the following sum:

$$\sum_{n=0}^{\infty} B_{2n} \frac{(2a_2)^{2n}(2k)!}{(2k-2n)!} \sum_{j=0}^{2k-2n} (-1)^{j-2n-1} \frac{2}{\sqrt{\pi}} H_{j-2n-1}\left(\frac{\gamma_1}{2a_2} \right) \exp\left(-\frac{\gamma_1^2}{4a_2^2} \right) \beta_{2k-2n,j}(\delta', \delta'', ..., \delta^{2k-2n-j}) +$$

$$+\sum_{n=0}^{\infty} B_{2n+1} \frac{(2a_2)^{2n+1}(2k)!}{(2k-2n-1)!} \sum_{j=0}^{2k-2n-1} (-1)^{j-2n} \frac{2}{\sqrt{\pi}} H_{j-2n-2}\left(-\frac{\gamma_1}{2a_2} \right) \exp\left(-\frac{\gamma_1^2}{4a_2^2} \right) \beta_{2k-2n-1,j}(\delta', \delta'', ..., \delta^{2k-2n-1-j}) = 0$$

Here let us denote the known coefficients in the following form:

$$K_{n,1}=\sum_{n=0}^{\infty}\frac{(2a_2)^{2n}(2k)!}{(2k-2n)!}\sum_{j=0}^{2k-2n}(-1)^{j-2n-1}\frac{2}{\sqrt{\pi}}H_{j-2n-1}\left(\frac{\gamma_1}{2a_2}\right)\exp\left(-\frac{\gamma_1^2}{4a_2^2}\right)\beta_{2k-2n,j}(\delta',\delta'',...,\delta^{2k-2n-j})$$

$$K_{n,2}=\sum_{n=0}^{\infty}\frac{(2a_2)^{2n+1}(2k)!}{(2k-2n-1)!}\sum_{j=0}^{2k-2n-1}(-1)^{j-2n}\frac{2}{\sqrt{\pi}}H_{j-2n-2}\left(-\frac{\gamma_1}{2a_2}\right)\exp\left(-\frac{\gamma_1^2}{4a_2^2}\right)\beta_{2k-2n-1,j}(\delta',\delta'',...,\delta^{2k-2n-1-j})$$

After we can easily find our unknown coefficient:

$$B_{2n}\cdot K_{n,1}=B_{2n+1}\cdot K_{n,2}$$

$$B_{2n}=\frac{B_{2n+1}\cdot K_{n,2}}{K_{n,1}}=\frac{f^{(2n+1)}(0)\cdot K_{n,2}}{2K_{n,1}} \tag{16}$$

From the condition (6) for $u_1(x,t)$ will be the same situation for $u_1(\alpha(t),t)=u_m$:

$$A_{2n}=\frac{A_{2n+1}\cdot K_{n,2}}{K_{n,1}} \tag{17}$$

By using the condition (7):

$$dt=2\tau d\tau$$

$$\left(\frac{L\gamma}{2\tau}\frac{d\alpha(\tau)}{d\tau}\right)\Bigg|_{\tau=0}^{(k)}=\left(\frac{L\gamma}{2}\sum_{n=0}^{\infty}\frac{\alpha^{(n)}(0)}{(2n)!}\tau^{2n-1}\right)\Bigg|_{\tau=0}^{(k)}$$

It is easy to see that $k=2n-1$, therefore:

$$\frac{L\gamma}{2}\sum_{n=0}^{\infty}\frac{\alpha^{(n)}(0)}{(2n)!}\left(\tau^{2n-1}\right)\Bigg|_{\tau=0}^{(k)}=\frac{L\gamma}{2}\sum_{n=0}^{\infty}\frac{\alpha^{(n)}(0)}{2n}$$

If we will take $2n-1$ derivatives in both parts by using Leibniz and Faa di Bruno formulas:

$$\left(-\lambda_1\frac{\partial u_1}{\partial x}\Big|_{x=\alpha(\tau^2)}\right)\Bigg|_{\tau=0}^{(2n-1)}=\left(-\lambda_2\frac{\partial u_2}{\partial x}\Big|_{x=\alpha(\tau^2)}\right)\Bigg|_{\tau=0}^{(2n-1)}+\frac{L\gamma}{4}\sum_{n=0}^{\infty}\frac{\alpha^{(n)}(0)}{n}$$

$$\left[\lambda_1\sum_{n=0}^{\infty}\left(2a_1\right)^{2n-1}A_{2n}(2n-1)!\left(i^{2n-1}erfc\left(\frac{\alpha(\tau^2)}{2a_1\tau}\right)\right)\Bigg|_{\tau=0}-\right.$$

$$\left.-\lambda_1\sum_{n=0}^{\infty}\left(2a_1\right)^{2n}A_{2n+1}\cdot\tau\cdot(2n-1)!\left(i^{2n}erfc\left(-\frac{\alpha(\tau^2)}{2a_1\tau}\right)\right)\Bigg|_{\tau=0}\right]=$$

$$\lambda_1\sum_{n=0}^{\infty}\left(2a_1\right)^{2n-1}A_{2n}(2n-1)!\left(i^{2n-1}erfc\left(\frac{\gamma_1+\gamma_2\tau+...}{2a_1}\right)\right)\Bigg|_{\tau=0}=A_{2n}\sum_{n=0}^{\infty}\lambda_1\left(2a_1\right)^{2n-1}(2n-1)!\left(i^{2n-1}erfc\left(\frac{\gamma_1}{2a_1}\right)\right)$$

$$\sum_{n=0}^{\infty}A_{2n}C_{n,1}=\sum_{n=0}^{\infty}B_{2n}C_{n,2}+\frac{L\gamma}{4}\sum_{n=0}^{\infty}\frac{\alpha^{(n)}(0)}{n} \tag{18}$$

where

$$C_{n,1} = \lambda_1 (2a_1)^{2n-1} (2n-1)! i^{2n-1} erfc\left(\frac{\gamma_1}{2a_1}\right)$$

$$C_{n,2} = \lambda_2 (2a_2)^{2n-1} (2n-1)! i^{2n-1} erfc\left(\frac{\gamma_1}{2a_2}\right)$$

From (17) and (18) we can easily find two unknowns in the following form:

$$A_{2n} = \frac{4n \cdot C_{n,2} \cdot f^{(2n+1)}(0) \cdot K_{n,2} + L\gamma \cdot \alpha^{(n)}(0) K_{n,1}}{8n \cdot K_{n,1} \cdot C_{n,1}} \tag{19}$$

$$A_{2n+1} = \frac{4n \cdot C_{n,2} \cdot f^{(2n+1)}(0) \cdot K_{n,2} + L\gamma \cdot \alpha^{(n)}(0) K_{n,1}}{8n \cdot K_{n,2} \cdot C_{n,1}} \tag{20}$$

By using the last condition (5) we get:

$$\lambda_1 \sum_{n=0}^{\infty} (2a_1)^{2n-1} \left[A_{2n} (\tau)^{2n-1} i^{2n-1} erfc(0) - A_{2n+1} (\tau)^{2n} i^{2n} erfc(0) \right] = \sum_{n=0}^{\infty} P_n \tau^n \tag{21}$$

The main problem is to find the remaining unknown coefficient we have:

$$\frac{\partial^k}{\partial \tau^k}\left(\lambda_1 \sum_{n=0}^{\infty} \left[\left(2a_1\tau\right)^{2n-1} A_{2n} i^{2n-1} erfc\left(\frac{x}{2a_1\tau}\right) + \left(2a_1\tau\right)^{2n} A_{2n+1} i^{2n} erfc\left(-\frac{x}{2a_1\tau}\right) \right] \right) = \frac{\partial^k}{\partial \tau^k}\left(\sum_{n=0}^{\infty} P_n \tau^n \right)$$

$$\lambda_1 \sum_{n=0}^{\infty} \sum_{l=0}^{k} \left[\left(2a_1\right)^{2n-1} A_{2n} \binom{k}{l} \frac{(2n-1)!}{(2n-1-k+l)!} \cdot \tau^{2n-1-k+l} \cdot \frac{\partial^k}{\partial \tau^k}\left(i^{2n-1} erfc\left(\frac{x}{2a_1\tau}\right)\right) - \right.$$

$$\left. -\left(2a_1\right)^{2n} A_{2n+1} \binom{k}{l} \frac{(2n)!}{(2n-k+l)!} \cdot \tau^{2n-k+l} \cdot \frac{\partial^k}{\partial \tau^k}\left(i^{2n} erfc\left(-\frac{x}{2a_1\tau}\right)\right) \right] = \frac{\partial^k}{\partial \tau^k}\left(\sum_{n=0}^{\infty} P_n \tau^n \right)$$

After taking both parts k-derivatives we can see following expression:

$$\lambda_1 \left[\sum_{n=0}^{\infty} \left(2a_1\right)^{2n-1} A_{2n} \frac{(k)!}{(k-2n+1)!} \frac{\partial^{k-2n+1}}{\partial \tau^{k-2n+1}} \left(i^{2n-1} erfc\left(\frac{x}{2a_1\tau}\right)\right)\right|_{\tau=0} -$$

$$-\left(2a_1\right)^{2n} A_{2n+1} \frac{(k)!}{(k-2n)!} \frac{\partial^{k-2n}}{\partial \tau^{k-2n}} \left(i^{2n} erfc\left(-\frac{x}{2a_2\tau}\right)\right)\Bigg|_{\tau=0} = \left(\sum_{n=0}^{\infty} P_n \frac{k!}{(k-n)!} \right)$$

For this purpose we use the Faa Di Bruno formula (Arbogast formula) for a derivative of a composite function:

$$\frac{\partial^{k-2n+1}}{\partial \tau^{k-2n+1}}\left(i^{2n-1} erfc\left(\frac{x}{2a_2\tau}\right)\right) = \sum_{j=0}^{k-2n+1} i^{2n-1} erfc^{(j)}(\delta) \beta_{k-2n+1,j}(\delta', \delta'', ..., \delta^{k-2n+1-j})$$

$$i^{2n-1} erfc^{(j)}(\delta) = \frac{d^j}{d\tau^j}\left(i^{2n-1} erfc(\delta)\right)\Big|_{\tau=0} = (-1)^{j-2n} \frac{2}{\sqrt{\pi}} H_{j-2n}(\delta) \exp(-\delta^2)$$

$$\delta\big|_{x=0} = \frac{x}{2a_2\tau}\bigg|_{x=0} = 0 \qquad \exp(0) = 1$$

$$\frac{\partial^{k-2n}}{\partial\tau^{k-2n}}\left(i^{2n}erfc\left(-\frac{x}{2a_2\tau}\right)\right) = \sum_{j=0}^{k-2n} i^{2n}erfc^{(j)}(\delta)\beta_{k-2n,j}(\delta',\delta'',...,\delta^{k-2n-j})$$

$$i^{2n}erfc^{(j)}(\delta) = \frac{d^j}{d\tau^j}\left(i^{2n}erfc(\delta)\right)\bigg|_{\tau=0} = (-1)^{j-2n-1}\frac{2}{\sqrt{\pi}}H_{j-2n-1}(\delta)\exp(-\delta^2)$$

Finally, we get following sum:

$$\lambda_1\left[\sum_{n=0}^{\infty} A_{2n}\frac{(2a_1)^{2n-1}(k)!}{(k-2n+1)!}\sum_{j=0}^{k-2n+1}(-1)^{j-2n}\frac{2}{\sqrt{\pi}}H_{j-2n}(0)\beta_{k-2n+1,j}(\delta',\delta'',...,\delta^{k-2n+1-j}) - \right.$$

$$\left. -\sum_{n=0}^{\infty} A_{2n+1}\frac{(2a_1)^{2n}(k)!}{(k-2n)!}\sum_{j=0}^{k-2n}(-1)^{j-2n-1}\frac{2}{\sqrt{\pi}}H_{j-2n-1}(0)\beta_{k-2n,j}(\delta',\delta'',...,\delta^{k-2n-j})\right] = \sum_{n=0}^{\infty} P_n\frac{k!}{(k-n)!}$$

After we can easily find our unknown coefficient:

$$A_{2n}\cdot K_{n,3} - A_{2n+1}\cdot K_{n,4} = P_n \tag{22}$$

1.3. Conclusion

Main result namely coefficients of function A_{2k}, A_{2k+1} (9), B_{2k}, B_{2k+1} (10) and the function $P(t)$ are obtained from (19), (20), (16), (15) and (21-22) respectively. It worth nothing that deviation of solution can provide maximum principle. Moreover if convergence of (9) and (10) proved, wide class of heat equations that describe diverse phenomena can be solved both analytically and numerically.

II. APPROXIMATE SOLUTION OF TWO PHASE INVERSE STEFAN PROBLEM

2.1. Introduction

Exact solutions of heat conduction problems are rather cumbersome and time-consuming. In addition, they are practically absent in the problems of the radial heat flux in spherical coordinates with the change in the aggregate state [11], 12]. Therefore, to solve practical problems are commonly used charts, obtained by numerical or approximate methods [13]. One of the approximate analytical methods is an integral method of heat balance, which primarily attracted to its physical clarity, simplicity and precision of the results is high enough that demonstrates T. Goodman [14] on numerous examples. The main difficulty to be faced when using the integral method of heat balance is the setting right temperature profile, which according to T. Goodman, significantly affects the accuracy of the results.

There are several approaches in selecting the temperature profiles. In [15] A.I. Veinik offers to use temperature profiles in the form of ordinary polynomials for problems of any geometry, which should simplify the solution of the problem.

In this study we will try to find approximate solution of two Phase Inverse Stefan problem for degenerate domain with moving boundary.

2.2. Problem Solution

Let us consider this problem at the suggestion that $\alpha(t) = \sum_{n=1}^{\infty} \alpha_n t^{n/2}$ is given.

To find the unknown $u_2(x,t)$, we identify the temperature profile corresponding following conditions:

$$u_2(\alpha(t),t) = u_m \tag{23}$$

$$u_2\big|_{\beta(t)} = 0 \tag{24}$$

$$\left.\frac{\partial u_2}{\partial x}\right|_{x=\beta(t)} = 0 \tag{25}$$

$$u_2\,|_{x=\infty} = 0 \tag{26}$$

Therefore the temperature profile will be:

$$u_2(x,t) = \begin{cases} u_m\left[\dfrac{x-\beta(t)}{\alpha(t)-\beta(t)}\right]^2, & \alpha(t) < x < \beta(t) \\ 0, & x > \beta(t) \end{cases} \tag{27}$$

Here we need to find the $\beta(t)$, where $\beta(0) = 0$ by using the equation (2) to use the integral of power balance:

$$\int_{\alpha(t)}^{\beta(t)} \frac{\partial u_2}{\partial t} dx = a_2^2 \frac{\partial u_2}{\partial x}\Big]_{\alpha(t)}^{\beta(t)} \tag{28}$$

$$\int_{\alpha(t)}^{\beta(t)} \frac{\partial u_2}{\partial t} dx = a_2^2 \left[\left.\frac{\partial u_2}{\partial x}\right|_{x=\beta(t)} - \left.\frac{\partial u_2}{\partial x}\right|_{x=\alpha(t)}\right]$$

$$\int_{\alpha(t)}^{\beta(t)} \frac{\partial u_2}{\partial t} dx = a_2^2 \left[0 - 2u_m\left[\frac{x-\beta(t)}{\alpha(t)-\beta(t)}\right]\left(\frac{1}{\alpha(t)-\beta(t)}\right)\right]_{x=\alpha(t)}$$

$$\int_{\alpha(t)}^{\beta(t)} \frac{\partial u_2}{\partial t} dx = -a_2^2 \frac{2u_m}{\alpha(t)-\beta(t)}$$

For the right side we use the Leibniz integral rule, also differentiation under the integral sign:

$$\frac{d}{dx}\left(\int_{\alpha(x)}^{b(x)} f(x,t)dt\right) = f(x,b(x))\cdot b'(x) - f(x,a(x))\cdot a'(x) + \int_{\alpha(x)}^{b(x)} \frac{\partial}{\partial x} f(x,t)dt$$

Therefore,

$$\int_{\alpha(t)}^{\beta(t)} \frac{\partial}{\partial t} f(x,t)dx = \frac{d}{dt}\left(\int_{\alpha(x)}^{\beta(x)} f(x,t)dx\right) - f(\beta(t),t)\cdot\beta'(t) + f(\alpha(t),t)\cdot\alpha'(t) \tag{29}$$

By using Leibniz's method we get:

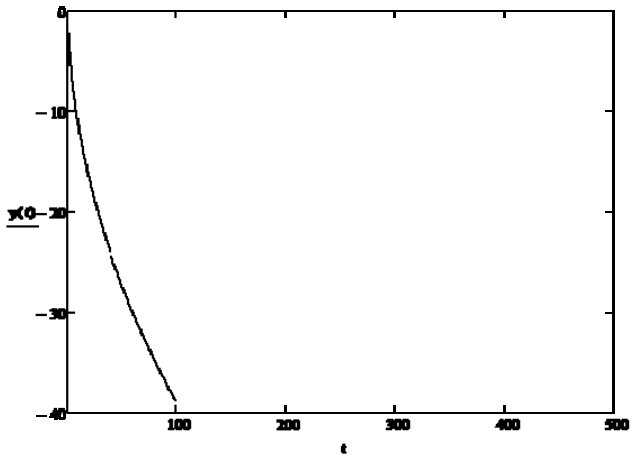
$$\left[\frac{d}{dt}\left(\int_{\alpha(t)}^{\beta(t)} u_2 dx\right) - u_2\,|_{x=\beta(t)}\,\frac{d\beta(t)}{dt} + u_2\,|_{x=\alpha(t)}\,\frac{d\alpha(t)}{dt}\right] = -a_2^2\,\frac{2u_m}{\alpha(t)-\beta(t)}$$

The solution of given differential equation has been shown in the paper [24].

$$\frac{d\beta(t)}{dt}\big(\beta(t)-\alpha(t)\big) + 2\frac{d\alpha(t)}{dt}\big(\beta(t)-\alpha(t)\big) - 6a_2^2 \tag{30}$$

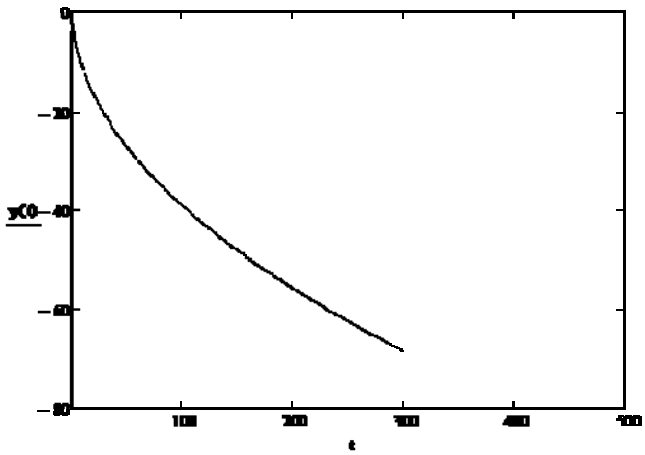
To solve the equation (30) we have used the mathematical program Mathcad 13. To find the solution we considered three particular cases and got following graphs:

$$\frac{d}{dt}y(t)\big(y(t)-\alpha(t)\big) + 2\frac{d\alpha(t)}{dt}\big(y(t)-\alpha(t)\big) - 6a_2^2 = 0 \tag{31}$$



Equation (31) have been plotted in the following way for the first particular case when $t = 100$:

Figure 1 - $\beta(t)$ at $t = 100$



Equation (31) have been plotted in the following way for the second particular case when $t = 300$:

Figure 2 - $\beta(t)$ at $t = 300$

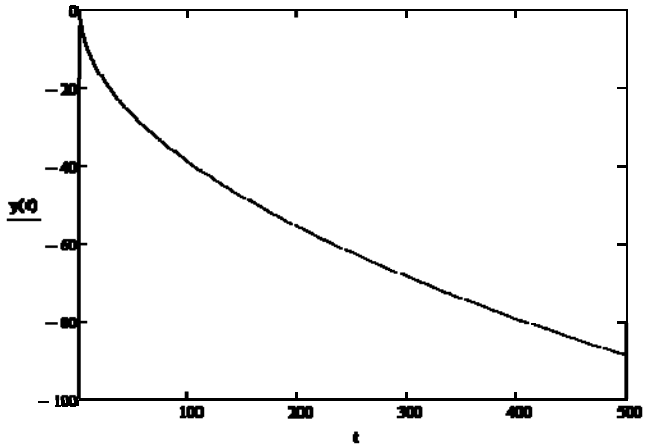


Figure 3 - $\beta(t)$ at $t = 500$

Equation (31) have been plotted in the following way for the third particular case when $t = 500$:

This given plots approximately show the value of $\beta(t)$. Therefore, we can consider that we know the value of $\beta(t)$ and consider other cases.

For $u_1(x,t)$ we have the temperature profile $A(t)x^2 + B(t)x + C(t)$ as it is linear. We will use the conditions (5),(6) for $u_1(x,t)$, (7) and (1):

$$u_1(x,t) = A(t)x^2 + B(t)x + C(t), \qquad 0 < x < \alpha(t)$$

By using the condition (5) we get:

$$\left.\frac{\partial u_1}{\partial x}\right|_{x=0} = B(t) = -\frac{P(t)}{\lambda_1} \tag{32}$$

By using the condition (6) for $u_1(x,t)$ we have:

$$u_1(\alpha(t),t) = A(t)\alpha^2(t) + B(t)\alpha(t) + C(t) = u_m \tag{33}$$

By using the Stefan condition (7) we get:

$$-\lambda_1(2A(t)\alpha(t) + B(t)) = -\lambda_2\left(\frac{2u_m}{\alpha(t) - \beta(t)}\right) + L\gamma\frac{d\alpha(t)}{dt}$$

If we equalize the right side of the equation above to $Q(t)$, we will have following expression:

$$2A(t)\alpha(t) + B(t) = -\frac{Q(t)}{\lambda_1} \tag{34}$$

Let us use the last equation (1) and get an integral of power balance:

$$\int_0^{\alpha(t)} \frac{\partial^2 u_1}{\partial x^2}\,dx = \left.\frac{\partial u_1}{\partial x}\right]_0^{\alpha(t)}$$

$$a_1^2\left[\left.\frac{\partial u_1}{\partial x}\right|_{x=\alpha(t)}-\left.\frac{\partial u_1}{\partial x}\right|_{x=0}\right]=\int_0^{\alpha(t)}\frac{\partial u_1}{\partial t}dx$$

$$a_1^2\left[2A(t)\cdot\alpha(t)+B(t)-B(t)\right]=a_1^2\left[2A(t)\cdot\alpha(t)\right]=\int_0^{\alpha(t)}\frac{\partial u_1}{\partial t}dx$$

The solutions has been shown in the paper [10].

$$A'(t)\left[2\alpha^2(t)+4\alpha'(t)\cdot\alpha(t)\right]+A(t)\left[4\alpha'(t)+4\left(\alpha''(t)\cdot\alpha(t)+\left(\alpha'(t)\right)^2\right)-12a_1^2\right]+$$
$$+\left[\frac{d}{dt}\left[\frac{3\alpha'(t)Q(t)}{\lambda_1}+\frac{3\alpha(t)Q'(t)}{\lambda_1}\right]-u_m\frac{\alpha'(t)}{\alpha(t)}\right]=0 \tag{35}$$

If we denote the functions that is multiplied to $A'(t),\quad A(t)$ and some functions by $Z_1(t), Z_2(t)$ and $Z_3(t)$ respectively, then we get:

$$A'(t)\cdot Z_1(t)+A(t)\cdot Z_2(t)+Z_3(t)=0$$

We can easily rewrite our differential equation in the following form:

$$y'(x)+y(x)\cdot S(x)+T(x)=0$$

By solving our differential equation by Integrating Factor method that has been shown in the paper [10], we can show the solution of givendifferential equation:

$$A(t)=-e^{-\int S(t)dt}\cdot\left(\int T(t)\cdot e^{\int S(t)dt}dt\right)+c\cdot e^{-\int S(t)dt} \tag{36}$$

where $S(t)=Z_2(t)/Z_1(t)$ and $T(t)=Z_3(t)/Z_1(t)$. Let us denote the right side of above expression as $K(t)$, therefore we can show the temperature profile for $u_1(x,t)$ and reconstruct the boundary function $P(t)$:

$$u_1(x,t)=K(t)x^2-\left[\frac{Q(t)}{\lambda_1}+2K(t)\alpha(t)\right]x+\left[u_m+K(t)\alpha^2(t)+\frac{Q(t)\alpha(t)}{\lambda_1}\right] \tag{37}$$

$$P(t)=Q(t)+\frac{2K(t)\alpha(t)}{\lambda_1} \tag{38}$$

2.4. Conclusion

In this chapter for solving problems of heat conduction in spherical geometry by integral method of power balance for the first time evaluated the effectiveness of the temperature type profiles: $T(r,t)=polynomial$ and $T(r,t)=polynomial/r$ with polynomials of degree n. The results show that in dealing with problems of thermal conductivity for different geometry approximate integral method of power balance is not always possible to achieve sufficient accuracy when using temperature profiles of the same type.

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НАҚТЫ ЖӘНЕ ЖУЫҚТАЛҒАН ЕКІ ФАЗАЛЫ КЕРІ СТЕФАН МӘСЕЛЕЛЕРІНІҢ ШЕШІМІ

Аннотация. Берілген жұмыстың негізгі идеясы, қозғалатын шекаралары бар нұқсанды өңір үшін екі фазалық кері стефан мәселенің нақты шешімдерін табу болып табылады.

Төменде осы мәселелерді жауап қадағалау ұйымдастырылады. Аналитикалық шешім табу үшін біз негізінен С.Н.Харин жұмыстарында ұсынған әдістері Фаа Ди Бруно формуласын интегралды қателіктер фунцияларына қолданамыз. Интегралды қателіктер функциясы және оның жаңа әдістерінің қасиеттірі де анық көрінген.

Стефан маселесі нақты шешім үшін күрделі болып табылады. Сондықтан, біз шамамен шешімін табу үшін, x және t арқылы интегралдап алынған дифференциалдық теңдеулерге температуралық профильдер тағайындаймыз. Бұл шамамен шешімін табу үшін С.Н. Харинның ұсынған әдістерін қолдана отырып, интегралды жылулық тепе-теңдік әдісін қолдындық.

Тірек сөздер: Интегралды функцияларқателіктер әдісі, Интегралды жылулық тепе-теңдік әдісі, Екі фазалы кері Стефан мәселесі.

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ТОЧНОЕ И ПРИБЛИЖЕННОЕ РЕШЕНИИ ДВУХФАЗАВОЙ ОБРАТНОЙ ЗАДАЧИ СТЕФАНА

Аннотация. Основная идея является нахождение точных решений двухфазной обратной задачи Стефана для вырожденной области с движущейся границей.Отслеживание ответов этих вопросов будут организованы следующим образом. Для нахождения аналитического решения мы в основном следуем методом, предложенным С.Н. Харин применяя формулу Фаа Ди Бруно для интегральных функций ошибок. В продолжении раздела интегральные функции ошибки и ее свойств, необходимых для разработки новых методов.

Задача Стефана очень сложна для точного решения. Поэтому мы пытаемся найти приближенное решение путем присвоения температурных профилей с изменением дифференциального уравнения на уравнения теплового баланса, которые получаются путем интегрирования по x и t. Для нахождения приближенного решения мы в основном следуем методом, предложенным С.Н. Харин, применения метода интегрального теплового баланса.

Ключевые слова: Метод интегрального функцияошибок, Метод интегрального теплового баланса, Двухфазова обратная задача Стефана.

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